

## Check Your Understanding

8. Which law is expressed by the equation  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ ?
  - a. This is Ohm's law.
  - b. This is Wien's displacement law.
  - c. This is Snell's law.
  - d. This is Newton's law.
9. Explain why the index of refraction is always greater than or equal to one.
  - a. The formula for index of refraction,  $n$ , of a material is  $n = \frac{\text{speed of light in a material}}{\text{speed of light in a vacuum}} = \frac{v}{c}$ , where  $v > c$ , so  $n$  is always greater than one.
  - b. The formula for index of refraction,  $n$ , of a material is  $n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}} = \frac{c}{v}$ , where  $c > v$ , so  $n$  is always greater than one.
  - c. The formula for index of refraction,  $n$ , of a material is  $n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}} = \frac{c}{v}$ , where  $c, v > 1$ , so  $n$  is always greater than one.
  - d. The formula for refractive index,  $n$ , of a material is  $n = \frac{1}{\frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}}} = \frac{1}{c/v}$ , where  $c < v < 1$ , so  $n$  is always greater than one.
10. Write an equation that expresses the law of refraction.
  - a.  $\frac{n_1}{n_2} = \frac{\sin \theta_1}{\sin \theta_2}$
  - b.  $\frac{n_2}{n_1} = \left( \frac{\sin \theta_2}{\sin \theta_1} \right)^2$
  - c.  $\frac{n_1}{n_2} = \left( \frac{\sin \theta_2}{\sin \theta_1} \right)^2$
  - d.  $\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}$

## 16.3 Lenses

### Section Learning Objectives

*By the end of this section, you will be able to do the following:*

- Describe and predict image formation and magnification as a consequence of refraction through convex and concave lenses, use ray diagrams to confirm image formation, and discuss how these properties of lenses determine their applications
- Explain how the human eye works in terms of geometric optics
- Perform calculations, based on the thin-lens equation, to determine image and object distances, focal length, and image magnification, and use these calculations to confirm values determined from ray diagrams

### Section Key Terms

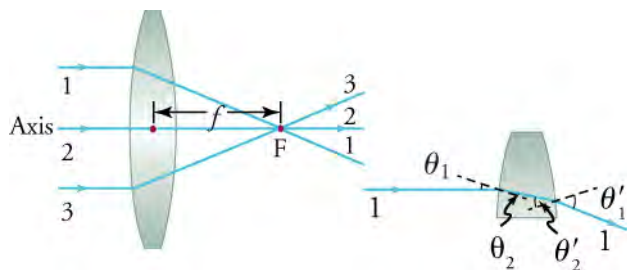
aberration	chromatic aberration	concave lens	converging lens	convex lens
diverging lens	eyepiece	objective	ocular	parfocal

### Characteristics of Lenses

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we use the law of refraction to explore the properties of lenses and how they form images.

Some of what we learned in the earlier discussion of curved mirrors also applies to the study of lenses. Concave, convex, focal point  $F$ , and focal length  $f$  have the same meanings as before, except each measurement is made from the center of the lens instead of the surface of the mirror. The **convex lens** shown in [Figure 16.25](#) has been shaped so that all light rays that enter it parallel to its central axis cross one another at a single point on the opposite side of the lens. The central axis, or axis, is defined to be a line normal to the lens at its center. Such a lens is called a **converging lens** because of the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown in [Figure 16.25](#) to illustrate how the ray changes

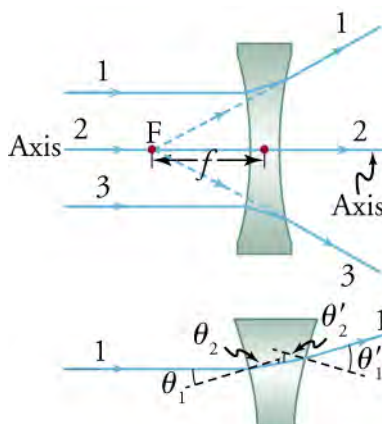
direction both as it enters and as it leaves the lens. Because the index of refraction of the lens is greater than that of air, the ray moves toward the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) As a result of the shape of the lens, light is thus bent toward the axis at both surfaces.



**Figure 16.25** Rays of light entering a convex, or converging, lens parallel to its axis converge at its focal point, F. Ray 2 lies on the axis of the lens. The distance from the center of the lens to the focal point is the focal length,  $f$ , of the lens. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Note that rays from a light source placed at the focal point of a converging lens emerge parallel from the other side of the lens. You may have heard of the trick of using a converging lens to focus rays of sunlight to a point. Such a concentration of light energy can produce enough heat to ignite paper.

[Figure 16.26](#) shows a **concave lens** and the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a **diverging lens** because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the *focal length*, or " $f$ ," of the lens. Note that the focal length of a diverging lens is defined to be negative. An expanded view of the path of one ray through the lens is shown in [Figure 16.26](#) to illustrate how the shape of the lens, together with the law of refraction, causes the ray to follow its particular path and diverge.



**Figure 16.26** Rays of light enter a concave, or diverging, lens parallel to its axis diverge and thus appear to originate from its focal point, F. The dashed lines are not rays; they indicate the directions from which the rays appear to come. The focal length,  $f$ , of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

The power,  $P$ , of a lens is very easy to calculate. It is simply the reciprocal of the focal length, expressed in meters

$$P = \frac{1}{f}.$$

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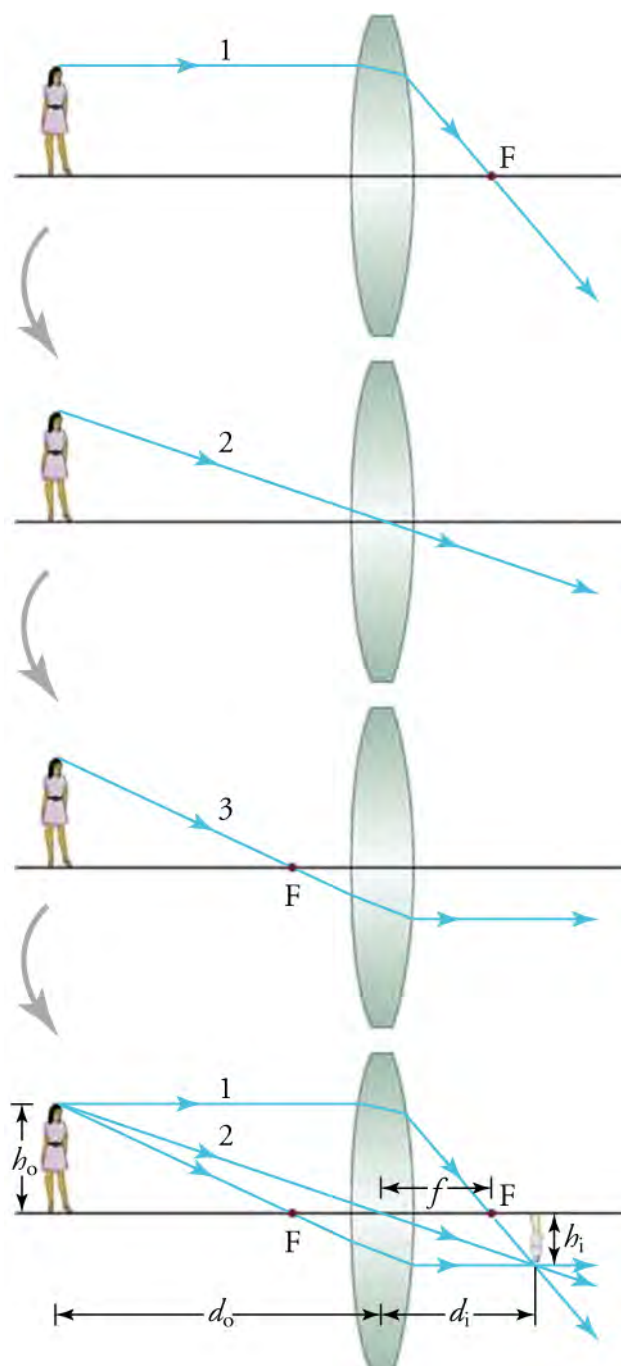
The units of power are diopters, D, which are expressed in reciprocal meters. If the focal length is negative, as it is for the diverging lens in [Figure 16.26](#), then the power is also negative.

In some circumstances, a lens forms an image at an obvious location, such as when a movie projector casts an image onto a screen. In other cases, the image location is less obvious. Where, for example, is the image formed by eyeglasses? We use ray

tracing for thin lenses to illustrate how they form images, and we develop equations to describe the image-formation quantitatively. These are the rules for ray tracing:

1. A ray entering a converging lens parallel to its axis passes through the focal point,  $F$ , of the lens on the other side
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point,  $F$ , on the side of the entering ray
3. A ray passing through the center of either a converging or a diverging lens does not change direction
4. A ray entering a converging lens through its focal point exits parallel to its axis
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis

Consider an object some distance away from a converging lens, as shown in [Figure 16.27](#). To find the location and size of the image formed, we trace the paths of select light rays originating from one point on the object. In this example, the originating point is the top of a woman's head. [Figure 16.27](#) shows three rays from the top of the object that can be traced using the ray-tracing rules just listed. Rays leave this point traveling in many directions, but we concentrate on only a few, which have paths that are easy to trace. The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). All rays that come from the same point on the top of the person's head are refracted in such a way as to cross at the same point on the other side of the lens. The image of the top of the person's head is located at this point. Rays from another point on the object, such as the belt buckle, also cross at another common point, forming a complete image, as shown. Although three rays are traced in [Figure 16.27](#), only two are necessary to locate the image. It is best to trace rays for which there are simple ray-tracing rules. Before applying ray tracing to other situations, let us consider the example shown in [Figure 16.27](#) in more detail.



**Figure 16.27** Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced. The three chosen rays each follow one of the rules for ray tracing, so their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen—is formed.

The image formed in [Figure 16.27](#) is a real image—meaning, it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye.

In [Figure 16.27](#), the object distance,  $d_o$ , is greater than  $f$ . Now we consider a ray diagram for a convex lens where  $d_o < f$ , and another diagram for a concave lens.

## Virtual Physics

### Geometric Optics

[Click to view content \(https://www.openstax.org/l/28Geometric\)](https://www.openstax.org/l/28Geometric)

This animation shows you how the image formed by a convex lens changes as you change object distance, curvature radius, refractive index, and diameter of the lens. To begin, choose Principal Rays in the upper left menu and then try varying some of the parameters indicated at the upper center. Show Help supplies a few helpful labels.

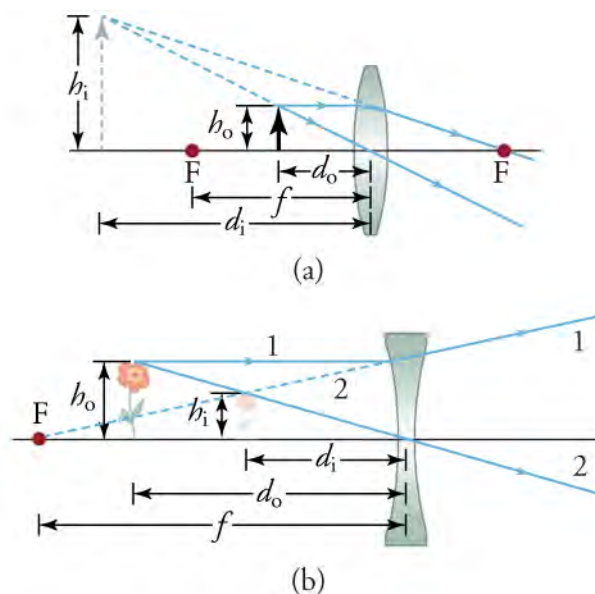
How does the focal length,  $f$ , change with an increasing radius of curvature? How does  $f$  change with an increasing refractive index?

- The focal length increases in both cases: when the radius of curvature and the refractive index increase.
- The focal length decreases in both cases: when the radius of curvature and the refractive index increase.
- The focal length increases when the radius of curvature increases; it decreases when the refractive index increases.
- The focal length decreases when the radius of curvature increases; it increases in when the refractive index increases.

Type	Formed When	Image Type	$d_i$	$M$
Case 1	$f$ positive, $d_o > f$	Real	Positive	Negative $m >$ , $<$ , or $= -1$
Case 2	$f$ positive, $d_o < f$	Virtual	Negative	Positive $m > 1$
Case 3	$f$ negative	Virtual	Negative	Positive $m < 1$

**Table 16.3** Three Types of Images Formed by Lenses

The examples in [Figure 16.27](#) and [Figure 16.28](#) represent the three possible cases—case 1, case 2, and case 3—summarized in [Table 16.3](#). In the table,  $m$  is magnification; the other symbols have the same meaning as they did for curved mirrors.



**Figure 16.28** (a) The image is virtual and larger than the object. (b) The image is virtual and smaller than the object.

## Snap Lab

### Focal Length

- Temperature extremes—Very hot or very cold temperatures are encountered in this lab that can cause burns. Use protective mitts, eyewear, and clothing when handling very hot or very cold objects. Notify your teacher immediately of any burns.
- EYE SAFETY—Looking at the Sun directly can cause permanent eye damage. Do not look at the Sun through any lens.
- Several lenses
- A sheet of white paper
- A ruler or tape measure

#### Instructions

#### Procedure

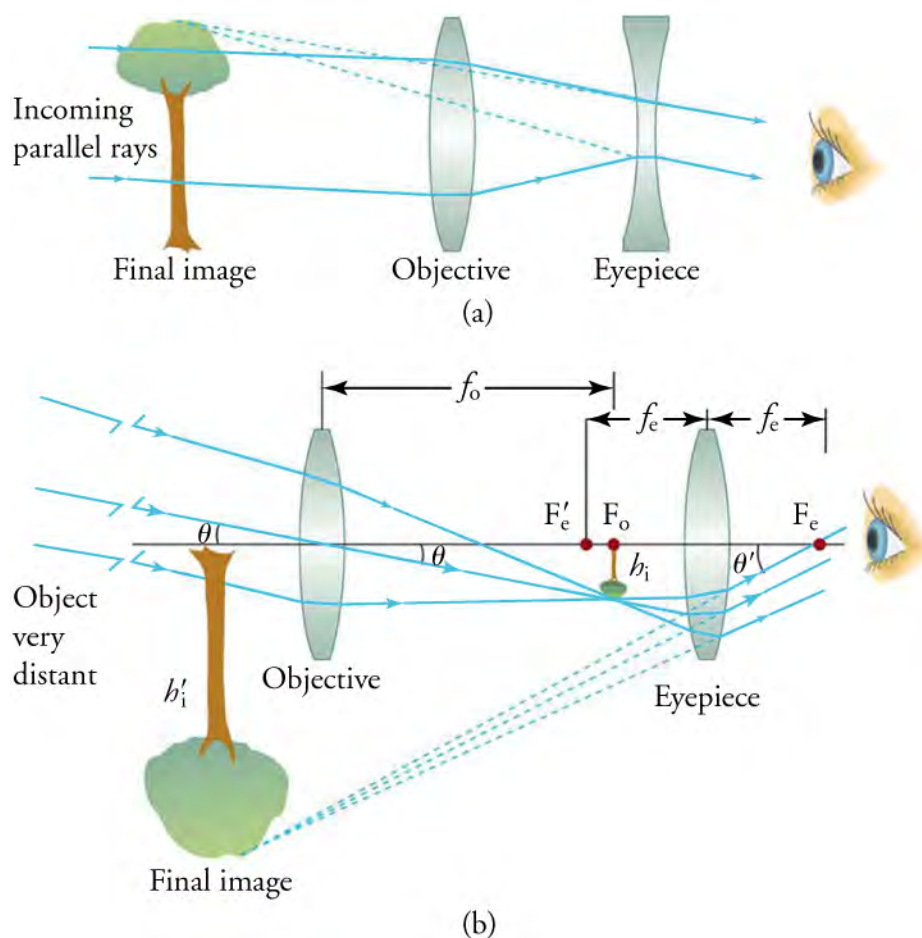
1. Find several lenses and determine whether they are converging or diverging. In general, those that are thicker near the edges are diverging and those that are thicker near the center are converging.
2. On a bright, sunny day take the converging lenses outside and try focusing the sunlight onto a sheet of white paper.
3. Determine the focal lengths of the lenses. Have one partner slowly move the lens toward and away from the paper until you find the distance at which the light spot is at its brightest. Have the other partner measure the distance from the lens to the bright spot. Be careful, because the paper may start to burn, depending on the type of lens.

True or false—The bright spot that appears in focus on the paper is an image of the Sun.

- a. True
- b. False

Image formation by lenses can also be calculated from simple equations. We learn how these calculations are carried out near the end of this section.

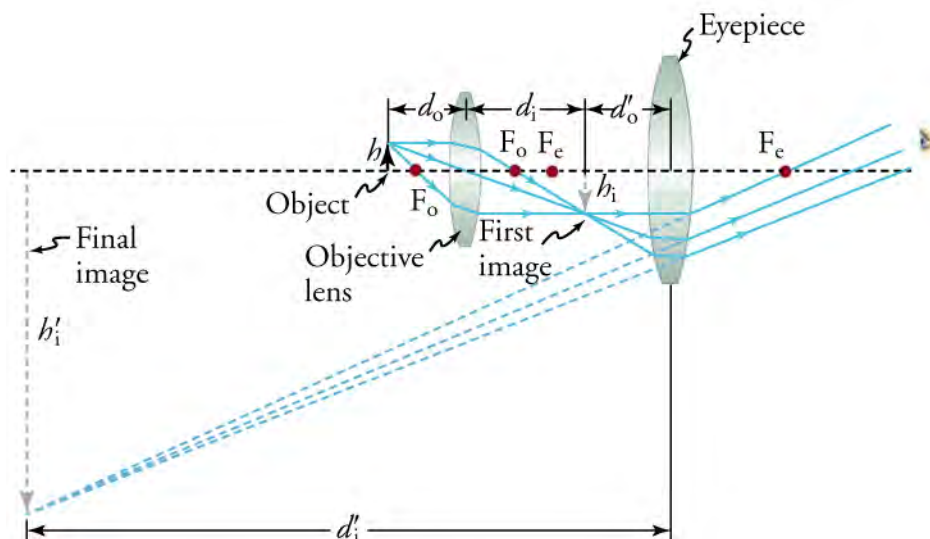
Some common applications of lenses with which we are all familiar are magnifying glasses, eyeglasses, cameras, microscopes, and telescopes. We take a look at the latter two examples, which are the most complex. We have already seen the design of a telescope that uses only mirrors in . [Figure 16.29](#) shows the design of a telescope that uses two lenses. Part (a) of the figure shows the design of the telescope used by Galileo. It produces an upright image, which is more convenient for many applications. Part (b) shows an arrangement of lenses used in many astronomical telescopes. This design produces an inverted image, which is less of a problem when viewing celestial objects.



**Figure 16.29** (a) Galileo made telescopes with a convex objective and a concave eyepiece. They produce an upright image and are used in spyglasses. (b) Most simple telescopes have two convex lenses. The objective forms a case 1 image, which is the object for the eyepiece. The eyepiece forms a case 2 final image that is magnified.

Figure 16.30 shows the path of light through a typical microscope. Microscopes were first developed during the early 1600s by eyeglass makers in the Netherlands and Denmark. The simplest compound microscope is constructed from two convex lenses, as shown schematically in Figure 16.30. The first lens is called the *objective lens*; it has typical magnification values from  $5\times$  to  $100\times$ . In standard microscopes, the **objectives** are mounted such that when you switch between them, the sample remains in focus. Objectives arranged in this way are described as **parfocal**. The second lens, the **eyepiece**, also referred to as the **ocular**, has several lenses that slide inside a cylindrical barrel. The focusing ability is provided by the movement of both the objective lens and the eyepiece. The purpose of a microscope is to magnify small objects, and both lenses contribute to the final magnification. In addition, the final enlarged image is produced in a location far enough from the observer to be viewed easily because the eye cannot focus on objects or images that are too close.

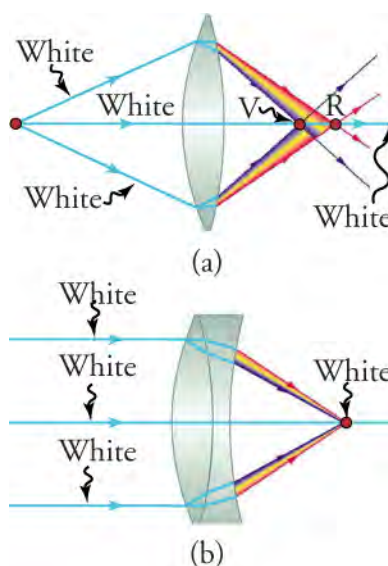




**Figure 16.30** A compound microscope composed of two lenses, an objective and an eyepiece. The objective forms a case 1 image that is larger than the object. This first image is the object for the eyepiece. The eyepiece forms a case 2 final image that is magnified even further.

Real lenses behave somewhat differently from how they are modeled using ray diagrams or the thin-lens equations. Real lenses produce aberrations. An **aberration** is a distortion in an image. There are a variety of aberrations that result from lens size, material, thickness, and the position of the object. One common type of aberration is **chromatic aberration**, which is related to color. Because the index of refraction of lenses depends on color, or wavelength, images are produced at different places and with different magnifications for different colors. The law of reflection is independent of wavelength, so mirrors do not have this problem. This result is another advantage for the use of mirrors in optical systems such as telescopes.

[Figure 16.31\(a\)](#) shows chromatic aberration for a single convex lens, and its partial correction with a two-lens system. The index of refraction of the lens increases with decreasing wavelength, so violet rays are refracted more than red rays, and are thus focused closer to the lens. The diverging lens corrects this in part, although it is usually not possible to do so completely. Lenses made of different materials and with different dispersions may be used. For example, an achromatic doublet consisting of a converging lens made of crown glass in contact with a diverging lens made of flint glass can reduce chromatic aberration dramatically ([Figure 16.31\(b\)](#)).

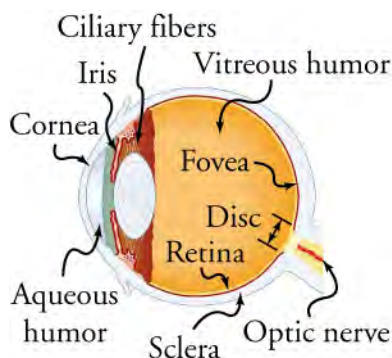


**Figure 16.31** (a) Chromatic aberration is caused by the dependence of a lens's index of refraction on color (wavelength). The lens is more powerful for violet (V) than for red (R), producing images with different colors, locations, and magnifications. (b) Multiple-lens systems can correct chromatic aberrations in part, but they may require lenses of different materials and add to the expense of optical systems such as cameras.



## Physics of the Eye

The eye is perhaps the most interesting of all optical instruments. It is remarkable in how it forms images and in the richness of detail and color they eye can detect. However, our eyes commonly need some correction to reach what is called *normal* vision, but should be called *ideal* vision instead. Image formation by our eyes and common vision correction are easy to analyze using geometric optics. [Figure 16.32](#) shows the basic anatomy of the eye. The cornea and lens form a system that, to a good approximation, acts as a single thin lens. For clear vision, a real image must be projected onto the light-sensitive retina, which lies at a fixed distance from the lens. The lens of the eye adjusts its power to produce an image on the retina for objects at different distances. The center of the image falls on the fovea, which has the greatest density of light receptors and the greatest acuity (sharpness) in the visual field. There are no receptors at the place where the optic nerve meets the eye, which is called the *blind spot*. An image falling on this spot cannot be seen. The variable opening (or pupil) of the eye along with chemical adaptation allows the eye to detect light intensities from the lowest observable to  $10^{10}$  times greater (without damage). Ten orders of magnitude is an incredible range of detection. Our eyes perform a vast number of functions, such as sense direction, movement, sophisticated colors, and distance. Processing of visual nerve impulses begins with interconnections in the retina and continues in the brain. The optic nerve conveys signals received by the eye to the brain.



**Figure 16.32** The cornea and lens of an eye act together to form a real image on the light-sensing retina, which has its densest concentration of receptors in the fovea, and a blind spot over the optic nerve. The power of the lens of an eye is adjustable to provide an image on the retina for varying object distances.

Refractive indices are crucial to image formation using lenses. [Table 16.4](#) shows refractive indices relevant to the eye. The biggest change in the refractive index—and the one that causes the greatest bending of rays—occurs at the cornea rather than the lens. The ray diagram in [Figure 16.33](#) shows image formation by the cornea and lens of the eye. The rays bend according to the refractive indices provided in [Table 16.4](#). The cornea provides about two-thirds of the magnification of the eye because the speed of light changes considerably while traveling from air into the cornea. The lens provides the remaining magnification needed to produce an image on the retina. The cornea and lens can be treated as a single thin lens, although the light rays pass through several layers of material (such as the cornea, aqueous humor, several layers in the lens, and vitreous humor), changing direction at each interface. The image formed is much like the one produced by a single convex lens. This result is a case 1 image. Images formed in the eye are inverted, but the brain inverts them once more to make them seem upright.

Material	Index of Refraction
Water	1.33
Air	1.00
Cornea	1.38
Aqueous humor	1.34

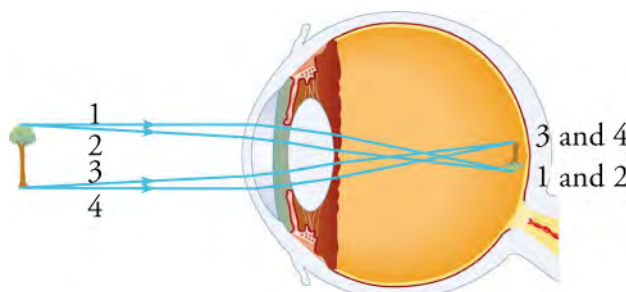
\*The index of refraction varies throughout the lens and is greatest at its center.

**Table 16.4** Refractive Indices Relevant to the Eye

Material	Index of Refraction
Lens	1.41 average*
Vitreous humor	1.34

\*The index of refraction varies throughout the lens and is greatest at its center.

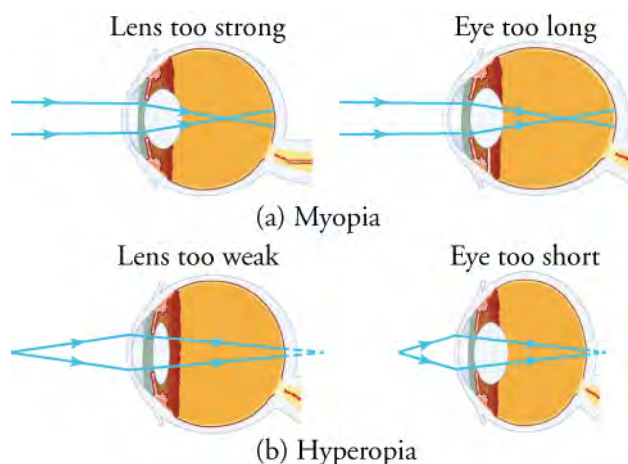
**Table 16.4** Refractive Indices Relevant to the Eye



**Figure 16.33** An image is formed on the retina, with light rays converging most at the cornea and on entering and exiting the lens. Rays from the top and bottom of the object are traced and produce an inverted real image on the retina. The distance to the object is drawn smaller than scale.

As noted, the image must fall precisely on the retina to produce clear vision—that is, the image distance,  $d_i$ , must equal the lens-to-retina distance. Because the lens-to-retina distance does not change,  $d_i$  must be the same for objects at all distances. The eye manages to vary the distance by varying the power (and focal length) of the lens to accommodate for objects at various distances. In [Figure 16.33](#), you can see the small ciliary muscles above and below the lens that change the shape of the lens and, thus, the focal length.

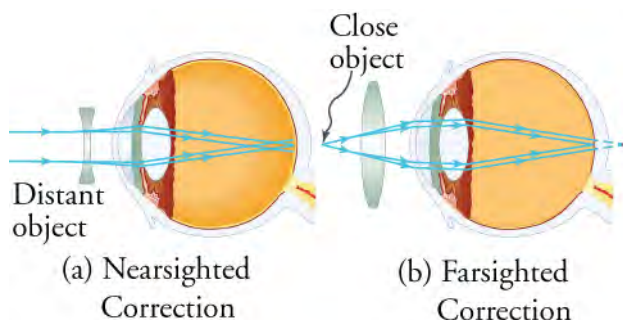
The need for some type of vision correction is very common. Common vision defects are easy to understand, and some are simple to correct. [Figure 16.34](#) illustrates two common vision defects. Nearsightedness, or myopia, is the inability to see distant objects clearly while close objects are in focus. The nearsighted eye *overconverges* the nearly parallel rays from a distant object, and the rays cross in front of the retina. More divergent rays from a close object are converged on the retina, producing a clear image. Farsightedness, or hyperopia, is the inability to see close objects clearly whereas distant objects may be in focus. A farsighted eye does not converge rays from a close object sufficiently to make the rays meet on the retina. Less divergent rays from a distant object can be converged for a clear image.



**Figure 16.34** (a) The nearsighted (myopic) eye converges rays from a distant object in front of the retina; thus, they are diverging when they strike the retina, and produce a blurry image. This divergence can be caused by the lens of the eye being too powerful (in other words, too short a focal length) or the length of the eye being too great. (b) The farsighted (hyperopic) eye is unable to converge the rays from a close object by the time they strike the retina and produce ... blurry close vision. This poor convergence can be caused by insufficient power (in

other words, too long a focal length) in the lens or by the eye being too short.

Because the nearsighted eye overconverges light rays, the correction for nearsightedness involves placing a diverging spectacle lens in front of the eye. This lens reduces the power of an eye that has too short a focal length (Figure 16.35(a)). Because the farsighted eye *underconverges* light rays, the correction for farsightedness is to place a converging spectacle lens in front of the eye. This lens increases the power of an eye that has too long a focal length (Figure 16.35(b)).



**Figure 16.35** (a) Correction of nearsightedness requires a diverging lens that compensates for the overconvergence by the eye. The diverging lens produces an image closer to the eye than the object so that the nearsighted person can see it clearly. (b) Correction of farsightedness uses a converging lens that compensates for the underconvergence by the eye. The converging lens produces an image farther from the eye than the object so that the farsighted person can see it clearly. In both (a) and (b), the rays that meet at the retina represent corrected vision, and the other rays represent blurred vision without corrective lenses.

## Calculations Using Lens Equations

As promised, there are no new equations to memorize. We can use equations already presented for solving problems involving curved mirrors. Careful analysis allows you to apply these equations to lenses. Here are the equations you need

$$P = \frac{1}{f},$$

where  $P$  is power, expressed in reciprocal meters ( $\text{m}^{-1}$ ) rather than diopters (D), and  $f$  is focal length, expressed in meters (m). You also need

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o},$$

where, as before,  $d_o$  and  $d_i$  are object distance and image distance, respectively. Remember, this equation is usually more useful if rearranged to solve for one of the variables. For example,

$$d_i = \frac{fd_o}{d_o - f}.$$

The equations for magnification,  $m$ , are also the same as for mirrors

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o},$$

where  $h_i$  and  $h_o$  are the image height and object height, respectively. Remember, also, that a negative  $d_i$  value indicates a virtual image and a negative  $h_i$  value indicates an inverted image.

These are the steps to follow when solving a lens problem:

- Step 1. Examine the situation to determine that image formation by a lens is involved.
- Step 2. Determine whether ray tracing, the thin-lens equations, or both should be used. A sketch is very helpful even if ray tracing is not specifically required by the problem. Write useful symbols and values on the sketch.
- Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).
- Step 4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is helpful to determine whether the situation involves a case 1, 2, or 3 image. Although these are just names for types of images, they have certain characteristics (given in Table 16.3) that can be of great use in solving problems.

- Step 5. If ray tracing is required, use the ray-tracing rules listed earlier in this section.
- Step 6. Most quantitative problems require the use of the thin-lens equations. These equations are solved in the usual manner by substituting knowns and solving for unknowns. Several worked examples were included earlier and can serve as guides.
- Step 7. Check whether the answer is reasonable. Does it make sense? If you identified the type of image (case 1, 2, or 3) correctly, you should assess whether your answer is consistent with the type of image, magnification, and so on.

All problems will be solved by one or more of the equations just presented, with ray tracing used only for general analysis of the problem. The steps then simplify to the following:

1. Identify the unknown.
2. Identify the knowns.
3. Choose an equation, plug in the knowns, and solve for the unknown.

Here are some worked examples:



## WORKED EXAMPLE

### The Power of a Magnifying Glass

#### Strategy

The Sun is so far away that its rays are nearly parallel when they reach Earth. The magnifying glass is a convex (or converging) lens, focusing the nearly parallel rays of sunlight. Thus, the focal length of the lens is the distance from the lens to the spot, and its power, in diopters (D), is the inverse of this distance (in reciprocal meters).

#### Solution

The focal length of the lens is the distance from the center of the lens to the spot, which we know to be 8.00 cm. Thus,

$$f = 8.00 \text{ cm.}$$

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To find the power of the lens, we must first convert the focal length to meters; then, we substitute this value into the equation for power.

$$P = \frac{1}{f} = \frac{1}{0.0800 \text{ m}} = 12.5 \text{ D}$$

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#### Discussion

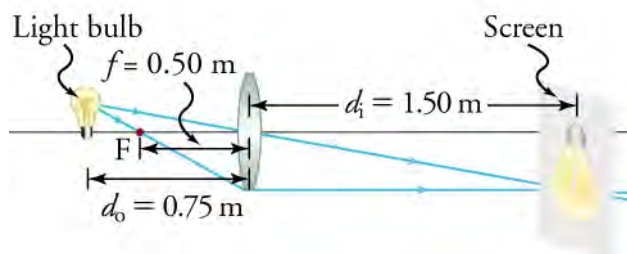
This result demonstrates a relatively powerful lens. Remember that the power of a lens in diopters should not be confused with the familiar concept of power in watts.



## WORKED EXAMPLE

### Image Formation by a Convex Lens

A clear glass light bulb is placed 0.75 m from a convex lens with a 0.50 m focal length, as shown in [Figure 16.36](#). Use ray tracing to get an approximate location for the image. Then, use the mirror/lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin-lens and magnification equations produce consistent results.



**Figure 16.36** A light bulb placed 0.75 m from a lens with a 0.50 m focal length produces a real image on a poster board, as discussed in the previous example. Ray tracing predicts the image location and size.

**Strategy**

Because the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to the one illustrated in the previous figure of a series of drawings showing a woman standing to the left of a lens. Ray tracing to scale should produce similar results for  $d_i$ . Numerical solutions for  $d_i$  and  $m$  can be obtained using the thin-lens and magnification equations, noting that  $d_o = 0.75$  m and  $f = 0.50$  m.

**Solution**

The ray tracing to scale in [Figure 16.36](#) shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus, the image distance,  $d_i$ , is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of two, and the image is inverted. Thus,  $m$  is about  $-2$ . The minus sign indicates the image is inverted. The lens equation can be rearranged to solve for  $d_i$  from the given information.

$$d_i = \frac{fd_o}{d_o - f} = \frac{(0.50)(0.75)}{0.75 - 0.50} = 1.5 \text{ m} \quad 16.18$$

Now, we use  $\frac{d_i}{d_o}$  to find  $m$ .

$$m = -\frac{d_i}{d_o} = -\frac{1.5}{0.75} = -2.0 \quad 16.19$$

**Discussion**

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the lens equation produce consistent results. The thin-lens equation gives the most precise results, and is limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you draw, but it is highly useful both conceptually and visually.

**WORKED EXAMPLE****Image Formation by a Concave Lens**

Suppose an object, such as a book page, is held 6.50 cm from a concave lens with a focal length of  $-10.0$  cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

**Strategy**

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is therefore the same, but the results are different in important ways.

**Solution**

$$d_i = \frac{fd_o}{d_o - f} = \frac{(-10.0)(6.50)}{6.50 - (-10.0)} = -3.94 \text{ cm} \quad 16.20$$

Now the magnification equation can be used to find the magnification,  $m$ , because both  $d_i$  and  $d_o$  are known. Entering their values gives

$$m = -\frac{d_i}{d_o} = -\frac{-3.94}{6.50} = 0.606. \quad 16.21$$

**Discussion**

A number of results in this example are true of all case 3 images. Magnification is positive (as calculated), meaning the image is upright. The magnification is also less than one, meaning the image is smaller than the object—in this case, a little more than half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. The image is virtual. The image is closer to the lens than the object, because the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, because the image is smaller than the object, you may think it is farther away; however, the image is closer than the object—a fact that is useful in correcting nearsightedness.



## WATCH PHYSICS

### The Lens Equation and Problem Solving

The video shows calculations for both concave and convex lenses. It also explains real versus virtual images, erect versus inverted images, and the significance of negative and positive signs for the involved variables.

[Click to view content \(https://www.openstax.org/l/28Lenses\)](https://www.openstax.org/l/28Lenses)

If a lens has a magnification of  $-\frac{1}{2}$ , how does the image compare with the object in height and orientation?

- The image is erect and is half as tall as the object.
- The image is erect and twice as tall as the object.
- The image is inverted and is half as tall as the object.
- The image is inverted and is twice as tall as the object.

### Practice Problems

- A lens has a focal length of 12.5 cm. What is the power of the lens?
  - The power of the lens is 0.0400 D.
  - The power of the lens is 0.0800 D.
  - The power of the lens is 4.00 D.
  - The power of the lens is 8.00 D.
- If a lens produces a 5.00 -cm tall image of an 8.00 -cm -high object when placed 10.0 cm from the lens, what is the apparent image distance? Construct a ray diagram using paper, a pencil, and a ruler to confirm your calculation.
  - 3.12 cm
  - 6.25 cm
  - 3.12 cm
  - 6.25 cm

### Check Your Understanding

- A lens has a magnification that is negative. What is the orientation of the image?
  - Negative magnification means the image is erect and real.
  - Negative magnification means the image is erect and virtual.
  - Negative magnification means the image is inverted and virtual.
  - Negative magnification means the image is inverted and real.
- Which part of the eye controls the amount of light that enters?
  - the pupil
  - the iris
  - the cornea
  - the retina
- An object is placed between the focal point and a convex lens. Describe the image that is formed in terms of its orientation, and whether the image is real or virtual.
  - The image is real and erect.
  - The image is real and inverted.
  - The image is virtual and erect.
  - The image is virtual and inverted.
- A farsighted person buys a pair of glasses to correct her farsightedness. Describe the main symptom of farsightedness and the type of lens that corrects it.
  - Farsighted people cannot focus on objects that are far away, but they can see nearby objects easily. A convex lens is used to correct this.
  - Farsighted people cannot focus on objects that are close up, but they can see far-off objects easily. A concave lens is used to correct this.

- c. Farsighted people cannot focus on objects that are close up, but they can see distant objects easily. A convex lens is used to correct this.
- d. Farsighted people cannot focus on objects that are either close up or far away. A concave lens is used to correct this.